**Multilayer Modularity Maximization using Leiden algorithm**

We have applied Multilayer Modularity maximization method using the Leiden solver to a several temporal network models where communities undergo transformations. We have created datasets from a continuous spike train data generated via Poisson process with fixed and varying uniformly random spike rates across layers. We have also chosen fixed and varying community sizes that are uniformly distributed across time. We have designated two types of community operations: merging and growth. These transformations also cover the cases where communities split and contract when time reversed. In this data set, communities undergo either of these transformations in a constant window size which we choose as the size of a window for every layer to maximize the correlation. Computing cross-correlation between pairs of neurons within each time window yield adjacency matrices which correspond to the layers of our networks.

Multilayer modularity maximization requires two parameters, interlayer coupling strength and a resolution parameter. In order to find the optimal set of parameters, we have run the algorithm on the grid [0,1] x [0,1] of step size 0.02, selecting a rough sub rectangle that gives the optimal partitions and running the Multilayer modularity on this sub rectangle with a consensus gave us the results we are looking for. Moreover, we’ve tried two different interlayer coupling where we’ve updated the interlayer edge weights according to the self-similarity of a node with it’s future self. We refer to these methods as local and global updates.

One significant flaw of the exploitation of a greedy solver is that MMM doesn’t know where to stop i.e. the less the number of communities the more the multilayer modularity score. This duality can be seen in our results. Nodes that does not belong to a significant community are grouped together to maximize modularity instead of being in smaller or individual communities. So, in this sense MMM creates false positives that makes impossible to interpret the results in the real world data because one can ask if the nodes in a community are forming actual communities or they are grouped together because they don’t belong to a community. One other disadvantage of using multilayer modularity as described in `Unspoken assumptions about MMM` is that the multilayer coupling strength is valid as much as the intralayer modularity score of within layer communities allow. The interlayer edge strength omega is a value that only rewards the optimal communities that extend across layers, however it does not punish the modularity score when the optimal communities are within layers—it’s effect to the modularity score only gets smaller yielding an increase in modularity.

The results yielded that multilayer modularity maximization is a reliable method to use if the communities